4-5 Determinants

The determinant of a matrix is a value of a square matrix that will be used to:
- calculate the inverse of a matrix
- solve systems

* find area of polygons

2x2 Matrix (2nd order determinants)
The determinant, D, is calculated as follows:

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

ex:

$$\begin{vmatrix} 5 & 10 \\ 8 & -3 \end{vmatrix} = -15 - 80 = -95$$

ex:

$$\begin{vmatrix} -2 & 6 \\ -5 & 1 \end{vmatrix} = -2 - -30 = 28$$

ex:

$$\begin{vmatrix} 4 & 10 \\ 2 & 5 \end{vmatrix} = 0$$

ex:

$$\begin{vmatrix} 4 & 6 \\ x & 12 \end{vmatrix} = 12$$

$$48 - 6x = 12$$

$$-6x = -36$$

$$x = 6$$

3x3 Matrix (3rd order determinants)
Befor calculating the determinant, D, we must define a minor.

Minor--of an element in a determinant is the determinant resulting from the deletion of the row and column containing the element.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix}$$

The minor of 5 is: $\begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}$

The minor of 6 is: $\begin{bmatrix} 5 & -2 \\ 2 & 4 \end{bmatrix}$

The minor of -7 is: $\begin{bmatrix} 5 & -1 \\ 2 & -3 \end{bmatrix}$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix}$$

Pick a row or column to expand

$$+5\begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -7 \\ 2 & 4 \end{vmatrix} + (-2)\begin{vmatrix} 3 & 6 \\ 2 & -3 \end{vmatrix}$$

$21 - 14 \qquad 12$

$3 \quad 24 \qquad 26 \quad 12 \qquad -9$

$15 \quad +26 \quad +42$

$D = 83$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 2 & -1 & -6 \\ 3 & 4 & 2 \\ 5 & -2 & 3 \end{vmatrix}$$

$= 187$

$$-3\begin{vmatrix} -1 & -6 \\ -2 & 3 \end{vmatrix} + 4\begin{vmatrix} 2 & -6 \\ 5 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$12 \qquad -30 \qquad -5$

$-3 \qquad 6 \qquad -4$

$45 + 144 - 2$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & 2 & 34 \\ -1 & 3 & 22 \\ 0 & 0 & 4 \end{vmatrix}$$

$$+4\begin{vmatrix} 5 & 2 \\ -1 & 3 \end{vmatrix}$$

-2

$= 68$ is

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15-19,27-31,39

Expansion by minors works for any higher order determinant as well.

ex:

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$